1. Grain Size Distribution Documentation of Calculations

The calculations used in the program are fairly simple and for the most part follow ASTM D 422. The following sections present the equations used.

1.1 Moisture Content

Moisture content is calculated with the following formula:

\[
MC = 100\% \times \frac{W_{wt} - W_{dt}}{W_{dt} - W_t}
\]  

Where:
- \( MC \) = the moisture content
- \( W_{wt} \) = the weight of the moisture specimen with tare
- \( W_{dt} \) = the weight of the dried specimen with tare
- \( W_t \) = the weight of the container

1.2 Sieve Test Calculations

1.2.1 Wash Test

\[
PF_{200} = \frac{W_{ts} - W_{wash}}{W_{ts}} \times 100\%
\]  

Where:
- \( PF_{200} \) = the percent of material finer than the #200 sieve
- \( W_{ts} \) = the total sample weight
- \( W_{wash} \) = the after-wash weight

The program charts \( PF_{200} \) as the #200 percentage if either no further sieve test data are entered (i.e., only a #200 wash test is performed), or the sieve test does not include a #200 sieve.
1.2.2 Cumulative Weight Retained Method

\[
PF = \left( 1 - \frac{W_c - W_{ct}}{W_{ts}} \right) \times 100\% \tag{1.3}
\]

Where:
- \(PF\) = the percent finer
- \(W_c\) = the cumulative weight retained
- \(W_{ct}\) = the tare weight of the cumulative pan
- \(W_{ts}\) = the total sample weight

An example calculation:
- Cumulative weight retained = 1915.2 grams
- Cumulative pan tare = 382.5 grams
- Sample weight = 1671.4 grams

\[
PF = \left( 1 - \frac{1915.2 - 382.5}{1671.4} \right) \times 100\% = 8.3\%
\]

1.2.3 Per-Sieve Weight Retained Method

\[
PF = 1 - \frac{(W_{st} - W_s) + W_{ls}}{W_{ts}} \times 100\% \tag{1.4}
\]

Where:
- \(W_{st}\) = the weight of a sieve and its retained material
- \(W_s\) = the weight of the sieve
- \(W_{ls}\) = the total amount of material retained on all larger sieves
- \(W_{ts}\) = the weight of the total sample

An example calculation:
- To calculate the percent finer for the third largest sieve in a sieve nest, we need:
  - Total sample weight = 11.94 grams
  - Weight retained + tare for the third largest sieve = 9.66 grams
  - Tare weight for the third largest sieve = 4.19 grams
  - Material retained on the larger sieves: 0.00 grams on the largest + 0.54 grams on the second largest.

\[
PF = 1 - \frac{(9.66 - 4.19) + 0.54}{11.94} \times 100\% = 49.7\%
\]
1.2.4 Sample Splits

If the sample is split, the subsequent percent finer values are found as follows:

\[ PF_{tot} = \frac{CBT - WR}{DWT} \]  \hspace{1cm} (1.5)

**Where:**
- \( PF_{tot} \) = the overall percent finer
- \( WR \) = the weight retained of the split sample

DWT, the post-split sample dry weight, is calculated as follows:

\[ DWT = \frac{SGDW}{PF_{ss} - PFW} \]  \hspace{1cm} (1.6)

**Where:**
- \( SGDW \) = the split gradation dry sample weight
- \( PF_{ss} \) = the overall percent finer than the split sieve
- \( PFW \) = the percent washed out from the #200 wash test (or 0 if a wash test was not performed)

CBT, the biased total weight, is calculated as follows:

\[ CBT = SGDW + PFW \times DWT \]  \hspace{1cm} (1.7)

**An example calculation:**
- \( PF_{ss} = 72.3\% \)
- \( SGDW = 1871.30 \) grams
- \( PFW = 0.00 \% \)
- \( WR = 422.00 \) grams

\[
DWT = \frac{1871.30}{72.30 - 0.00} = 25.90
\]

\[ CBT = 1871.30 + (0.00 \times 25.9) = 1871.30 \]

\[ PF = \frac{1871.30 - 442.00}{25.90} = 55.2\% \]
1.3 Hydrometer Test Calculations

1.3.1 Particle Size

\[ PS = \sqrt{\frac{30 \times v \times l}{980 \times (GS - GW) \times ET}} \]  \hspace{1cm} (1.8)

Where:

- \( PS \) = the particle size in mm.
- \( v \) = the fluid viscosity in centipoises
- \( l \) = the effective depth in cm.
- \( GS \) = the specific gravity of the soil particles
- \( GW \) = the specific gravity of water, corrected for temperature
- \( ET \) = the elapsed time in minutes

For 152H hydrometers, \( L \), the effective depth, is calculated as follows:

\[ L = 16.295 - 0.165 \times Rm \]  \hspace{1cm} (1.9)

\[ Rm = R - |mh| \]  \hspace{1cm} (1.10)

Where:

- \( Rm \) = the hydrometer reading corrected for the height of the meniscus.
- \( R \) = the hydrometer reading, taken at the top of the meniscus. (Alt., the hydrometer reading taken at the bottom of the meniscus if the meniscus correction is entered as 0.)
- \( mh \) = the meniscus height

The absolute value accounts for the fact that some users enter the value as a negative correction constant, while others enter it as the height of the meniscus.

For 151H hydrometers \( L \) is calculated with this equation:

\[ L = 16.295 - 0.2645 \times Rm \]  \hspace{1cm} (1.11)

\( \Rightarrow \) Note that the effective depth equation can be changed by the user on a per-test basis.

\( v \), the fluid viscosity, is calculated as:

\[ v = C1 + T \times (C2 + T \times (C3 + T \times (C4 + T \times C5))) \]  \hspace{1cm} (1.12)

Where:

- \( T \) = the fluid temperature, in degrees Celsius
- \( C1 = 0.01732483379693 \)
- \( C2 = -5.041574656095E-04 \)
- \( C3 = 8.387438669317E-06 \)
- \( C4 = -7.401129271698E-08 \)
- \( C5 = 2.625994080072E-10 \)
GW, the specific gravity of water, is calculated as:

\[
GW = C1 + T \times (C2 + T \times (C3 + T \times C4))
\]

(1.13)

Where:
- \(T\) = the fluid temperature, in degrees Celsius
- \(C1 = 0.99991003252\)
- \(C2 = 0.00005201921\)
- \(C3 = -0.00000751229\)
- \(C4 = 0.00000003605183\)

An example calculation:
- \(ET = 8\) minutes
- \(Temp = 23.5\) Celsius
- \(R = 34\)
- \(Cm = 1\)
- \(GS = 2.7\)
- \(Rm = 35\) (= \(R + Cm\))
- \(GW = 0.997452\) (calculation not shown)
- \(v = 0.00925\) (calculation not shown)
- \(L = 10.56\) (calculation not shown)

\[
PS = \sqrt{\frac{30 \times 0.00925 \times 10.56}{980 \times (2.7 - 0.9975) \times 8}} = 0.0148\ mm.
\]
1.3.2 Percent Finer

For 152H hydrometers, the percent finer than a given opening size is calculated as:

\[ PF = \frac{Rc \times a}{WB} \times 100\% \] (1.14)

Where:
- \( PF \) = the percent finer
- \( Rc \) = the corrected hydrometer reading
- \( a \) = the specific gravity of solids correction factor
- \( WB \) = the biased hydrometer sample weight

For 151H hydrometers, the calculation is:

\[ PF = \frac{100 \times GS}{WB \times (GS - 1)} \times Rc \] (1.15)

Where:
- \( PF \) = the percent finer
- \( Rc \) = the corrected hydrometer reading
- \( WB \) = the biased hydrometer sample weight
- \( GS \) = the soil specific gravity

When using automatic temperature correction, the corrected hydrometer reading (\( Rc \)) is calculated as follows:

\[ Rc = R + Ct + Cc \] (1.16)

Where:
- \( R \) = the actual hydrometer reading (in thousandths for 151H)
- \( Cc \) = the composite correction at 20 degrees Celsius, as entered by the user
- \( Ct \) = the composite correction

For 152H hydrometers, \( Ct \), the composite correction, is calculated as follows:

\[ Ct = -12.35952257 + T \times (1.51062059 + T \times (-0.06923056 + T \times 0.00122483)) \] (1.17)

Where:
- \( T \) = the fluid temperature, in degrees Celsius

For 151H hydrometers, \( Ct \) is calculated as:

\[ Ct = -7.6338851 + T \times (0.93361976 + T \times (-0.04284159 + T \times 0.000758977)) \] (1.18)

Where:
- \( T \) = the fluid temperature, in degrees Celsius
When using the multi-point (linear) temperature correction, the corrected hydrometer reading is calculated as follows:

\[ R_c = R + C_t \]  

(1.19)

**Where:**

- \( R \) = the actual hydrometer reading (in thousandths for 151H)
- \( C_t \) = the temperature correction, as interpolated from a linear regression line constructed from the pairs of temperature and reading values entered by the user into the hydrometer correction grid.

- For test temperatures less than the lowest temperature entered into the correction grid, the program will use the correction value corresponding to the lowest correction temperature entered; likewise, for test temperatures higher than the highest temperature entered into the correction grid, the program will use the correction value corresponding to the highest correction temperature entered.

The specific gravity correction factor is:

\[ a = \frac{0.6226415 \times GS}{GS - 1} \]  

(1.20)

**Where:**

- \( GS \) = the specific gravity of the solids

The biased sample weight is calculated as:

\[ WB = \frac{Whs \times 10000}{Pss \times (100 + Mh)} \]  

(1.21)

**Where:**

- \( WB \) = the biased sample weight, in grams
- \( Whs \) = the air dry hydrometer sample weight
- \( PSS \) = the percent passing the separation sieve
- \( Mh \) = the hygroscopic moisture content per ASTM D 422 § 8

**An example calculation using a 152H hydrometer:**

- \( ET = 8 \) minutes
- \( Temp. = 23.5 \) degrees Celsius
- \( R = 34 \)
- \( GS = 2.7 \)
- \( Mh = 3.5\% \)
- Linear correction pairs: (-6.0 at 20°), (-5.6 at 22°), (-4.7 at 25°)

By interpolation, \( Cc \) at 23.5° = -5.15
\[ Rc = 34 + (-5.15) = 28.9 \]

\[ a = \frac{0.6226415 \times 2.7}{2.7 - 1} = 0.989 \]

\[ WB = \frac{51.7 \times 10000}{100 \times (100 + 3.5)} = 50.0 \text{ grams} \]

\[ PF = \frac{28.9 \times 0.989}{50.0} = 57.0\% \]

**An additional calculation, using a 151H hydrometer:**

- ET = 15 minutes
- Temp = 22 Celsius
- R = 21.5
- GS = 2.65
- WB = 63.5 grams
- Cc at 22 degrees Celsius = -2.2

\[ Rc = 21.5 + (-2.2) = 19.3 \]

\[ PF = \frac{100 \times 2.65}{63.5 \times 1.65} \times 19.3 = 48.8\% \]
1.3.3 Calculation of Fractional Components

The fractional components and percentage diameters ($D_{85}$, $D_{60}$,$D_{50}$, etc.) are computed by creating a cubic spline model of the particle size distribution curve then solving the model for the curve values at various percentages.

The classification coefficients $C_c$ and $C_u$ are calculated as follows:

$$C_c = \frac{D_{30} \times D_{30}}{D_{60} \times D_{10}}$$

(1.22)

$$C_u = \frac{D_{60}}{D_{10}}$$

(1.23)

1.3.4 Tables of Constants

Table 1.1: Correction Factor for Specific Gravities Other than 2.65 when using Hydrometer 152H

<table>
<thead>
<tr>
<th>SPECIFIC GRAVITY</th>
<th>CORR. FACTOR</th>
<th>SPECIFIC GRAVITY</th>
<th>CORR. FACTOR</th>
</tr>
</thead>
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<td>2.50</td>
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Table 1.2: Automatic Temperature Correction Factor as a Function of Temperature

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<th>TEMP, DEG C.</th>
<th>SPECIFIC GRAVITY OF WATER</th>
<th>VISCOSITY OF WATER</th>
<th>TEMP. CORR. FACTOR FOR 152H</th>
<th>TEMP. CORR. FACTOR FOR 151H</th>
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</thead>
<tbody>
<tr>
<td>15.0</td>
<td>0.99912</td>
<td>0.01141</td>
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<td>-0.71</td>
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<td>-1.02</td>
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<td>-0.55</td>
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<tr>
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Table 1.3: Effective Depth for 152H and 151H Hydrometers

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<th>EFFECTIVE DEPTH</th>
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